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Peletier, R. F.; Grijs, R. de

Published in:
Default journal

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1997

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):
Peletier, R. F., & Grijs, R. D. (1997). Stellar Distributions and NIR Colours of Normal Galaxies. Default journal.

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STELLAR DISTRIBUTIONS AND NIR COLOURS OF NORMAL GALAXIES

R.F. Peletier ¹, R. de Grijs ¹

¹ *Kapteyn Astronomical Institute, Groningen, The Netherlands*



Abstract

We discuss some results of a morphological study of edge-on galaxies, based on optical and especially near-infrared surface photometry. We find that the vertical surface brightness distributions of galaxies are fitted very well by exponential profiles, much better than by isothermal distributions. We find that in general the vertical scale height increases when going outward. This increase is strong for early-type spiral galaxies and very small for late types. We argue that it can be due to the presence of thick discs with scale lengths larger than the galaxy's main disc. Finally we discuss the colour-magnitude relation in $I - K$ for spiral galaxies. We find that it is a tight relation, for which the scatter is similar to the observational uncertainties, with a steeper slope than for elliptical galaxies.

1 Introduction

To learn more about the processes responsible for the formation and evolution of galaxies one can either observe galaxies at high redshift to directly measure the evolution, or study nearby galaxies in more detail, to look for remnants of the formation process. In this paper we do the latter, and discuss some aspects of the morphology and stellar populations of spiral galaxies. This subject is not new. About 20 years ago the morphology of spiral galaxies was studied extensively by van der Kruit & Searle [31, 32, 33, 34]. At that time knowledge about stellar populations had come mainly from optical and near-infrared colour studies (e.g. [35, 1] and others). In the meantime, however, much better instruments and detectors have become

Invited Review to appear in “*Extragalactic Astronomy in the Infrared*”, eds. G. A. Mamon, Trinh Xuân Thuận, & J. Trần Thanh Vân, Editions Frontières, Gif-sur-Yvette

available, requiring these problems to be attacked again. Here (Section 2) we will discuss a morphological study in B , I and K of a large sample of edge-on spirals. The infrared array observations allow us to look through the dust, so that the vertical profiles near the symmetry plane of the galaxy can be studied much better than before. Away from the plane our deep observations allow us to address the question whether thick discs are common in spiral galaxies.

Although optical and near-infrared colours of galaxies have been studied frequently, their interpretation has been difficult as a result of extinction by dust. Only now infrared array observations have made it possible to obtain high-resolution optical and infrared maps, allowing people to choose areas that are not affected by dust, and in this way really study the stellar populations. In Section 3 we will discuss one of these dustfree colour- magnitude (CM) diagrams and investigate its implications.

2 Morphology of discs and bulges

2.1 Vertical profiles near the plane

Van der Kruit & Searle [31, 32, 33, 34] found that vertical profiles of spiral galaxies resemble self-gravitating isothermal sheets with the following light distribution:

$$K(z) = K_0 \operatorname{sech}^2(z/z_0) \quad (1)$$

where $K(z)$ is the observed vertical distribution, and z_0 the vertical scale parameter. At large z this distribution goes asymptotically to an exponential distribution. Their result was based on photographic observations of edge-on galaxies, which were severely affected by extinction. Later, more observations, especially in the near-infrared, became available (e.g. [36, 4]), showing that for the nearby galaxies IC 2531 and NGC 891 much better fits were obtained with purely exponential functions. Up to now this problem has not been studied for a large sample of edge-on galaxies. The problem with the exponential distribution is that it not understood physically, and that it implies very low velocity dispersions in the plane of the galaxy [30]. Seeing both the exponential and the isothermal distributions as extremes, he proposed to use a family of density laws:

$$K(z) = 2^{-2/n} K_0 \operatorname{sech}^{2/n}(nz/2z_0), \quad (n > 0) \quad (2)$$

For $n=1$ we have the sech^2 law (isothermal sheet), while for $n=\infty$ we have the exponential distribution. In Fig. 1a a family of these density laws is shown.

To determine for which n the best fits of eqn. (2) could be made to real spiral galaxies, we have observed a sample of 24 galaxies in K' [37] (calibrated to K), I and B . It is a subset of a complete diameter-limited sample of edge-on galaxies with inclinations larger than 87° , comprising galaxy types from S0 to Sd. Details about the observations are given in [15]. The observations have been taken at ESO, La Silla, from 1993 to 1996. The infrared observations were taken with IRAC2b on the 2.2m, while the optical observations came from the 1.5m Danish and the 0.9m Dutch telescope. The observations were taken under photometric conditions, and calibrated using standard stars, giving an accuracy of ≈ 0.08 mag in K' and 0.04 mag in I and B . The seeing ranged between $1''$ and $1.5''$. In Fig. 2 we plot some vertical profiles through the center in K' and I . It is clear that the profiles in the I -band near the symmetry plane are still significantly affected by extinction, while the symmetry in K' shows that extinction here barely affects the profiles. In the bottom panels the $I - K'$ profiles give a good idea of the amount of extinction near the mid-plane, assuming that $I - K'$ is constant across the profile. This

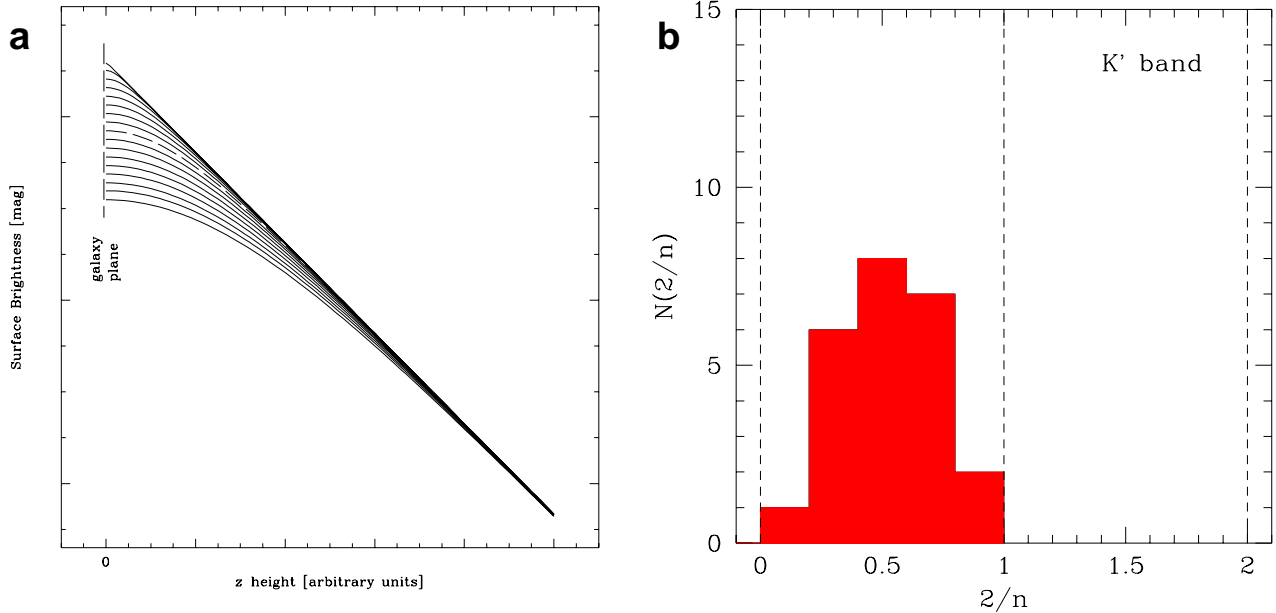


Figure 1: a: The family of density laws, with the isothermal ($2/n=1$) and the exponential ($2/n=0$) distributions as two extremes. The stepsize in n is 0.125; for clarity the sech(z) model ($2/n=1$) is shown with dashed lines. b: Histogram of average values of $(2/n)$ obtained from the extinction-corrected K' -band observations.

assumption is justified to a large extend by the fact that in the area where $I - K$ is a smooth function of position the change in colour is small, and in the central regions, where a lot of dust extinction is observed, the changes are large and the profiles often asymmetric. Moreover, stellar population models of old stellar populations predict very small changes in $I - K'$ (see e.g. [29]). For each profile we determined a constant, dust-free colour, used it to determine E_{I-K} at $z=0$, and corrected our K' using the Galactic extinction law [25] for residual extinction. After this we fitted eqn. (2) to all the corrected K' profiles, at various radial distances in the galaxy. In general the fits were acceptable, and in Fig. 1b we give the histogram of average n -values per galaxy. The radial distributions of n are given in Fig. 3. For signal-to-noise reasons we have binned together the data for galaxies of similar morphological types.

We see that the observed vertical profiles of all spiral galaxies have shapes that lie between the exponential and sech(z) distributions. Also, no variations as a function of radius along the major axis are seen. We now consider what this implies for the three-dimensional, deprojected profiles. If galaxies have an orientation of exactly 90 degrees, with everywhere the same vertical profile shape, projection along the line of sight leaves the profile shape invariant. However, at positions only a few degrees away from edge-on, the profiles flatten near $z=0$, severely affecting the determination of n . Simulations (described in [15]) show that at $i=87^\circ$ an exponential profile will look like a profile with $(2/n) = 0.7$ in projection, also somewhat dependent on position in the galaxy. Because of the uncertainties in the determination of the inclinations, we can only say that the average correction that we have to apply to $(2/n)$ lies between 0.3 and 0.5. Since we find (Fig. 1b) that $n = 0.54 \pm 0.2$ projected onto the plane of the sky, the deprojected profiles are probably very close to exponential ($n \approx 0.0-0.2$).

This result agrees very well with observations of our own galaxy. From near-infrared maps of the sky Kent *et al.* [20] showed that the best fits to our Galaxy could be obtained using an exponential distribution with radially increasing scale length. Similarly Kuijken & Gilmore [21]

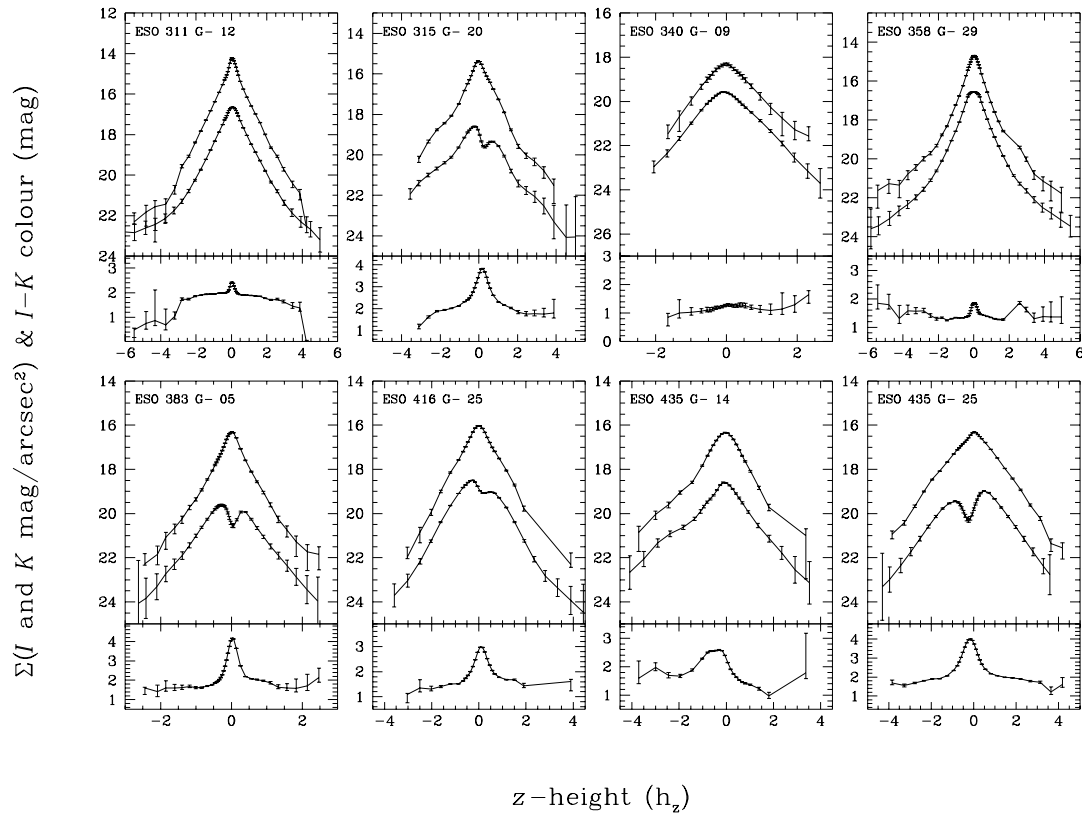


Figure 2: Central I and K' band vertical profiles of 8 galaxies, and their corresponding $I - K$ colours.

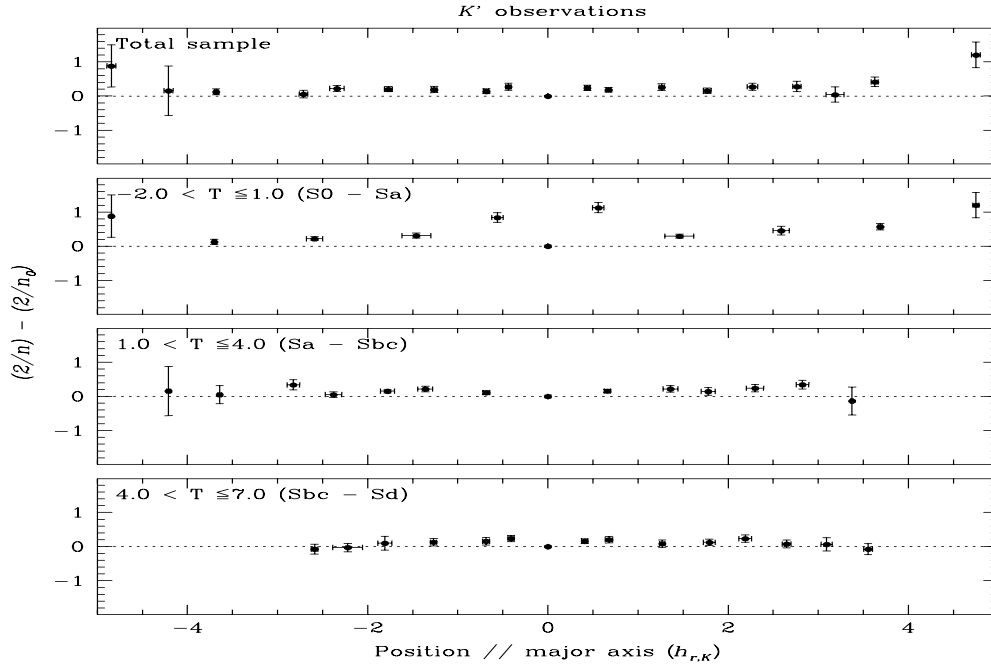


Figure 3: Averaged distributions of the sharpness parameter as a function of position along the major axis, both for the total sample and for specific Hubble type bins.

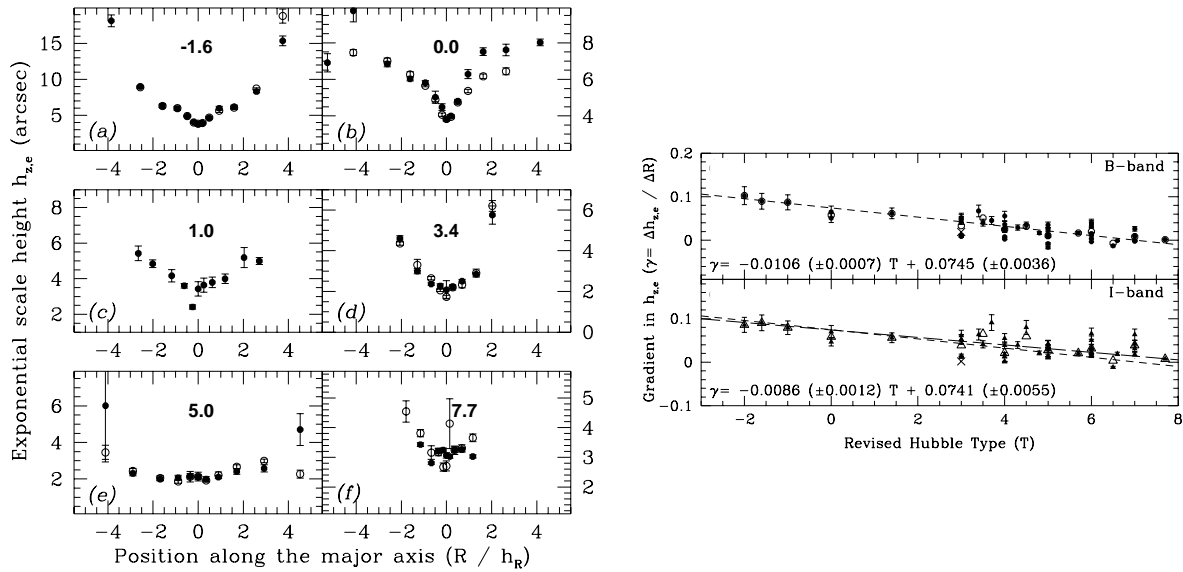


Figure 4: a: Examples of *I*-band scale height as a function of galactocentric distance for 6 galaxies of our sample. Morphological types are shown. Open and closed symbols represent data taken on both sides of the galaxy plane. b: Disc scale height gradients as a function of Hubble type. The closed symbols represent the data; the open symbols are type-averaged data points. The result that we obtained using van der Kruit & Searle [32]’s data for NGC 891 is indicated with crosses (\times). For comparison, the best fit obtained for the *B*-band data is also shown in the *I*-band figure.

found that the density profile of stars in the solar neighbourhood is also very close to exponential. Currently there are no simple models leading to exponential vertical profiles. Simulations of disc heating by spiral arms by Jenkins & Binney [18] produce a remarkably isothermal distribution of the stars in the solar neighbourhood. One way to make these distributions more peaked is to add new bursts of star formation. New simulations will have to answer the question of how much recent star formation for all spiral galaxies is implied by these observations.

2.2 Vertical profiles at large z -distances

In this subsection we analyze the outer parts of the vertical profiles, now using the *I*-band observations, since the *K'* band observations do not go deep enough due to the high infrared sky background. Studies of edge-on galaxies have generally shown that away from the central plane the light distribution can be fitted well by an exponential distribution, for which the radial variation of the scale height is typically within 3% of the mean [31, 32, 33, 34, 26, 6]. The fact that the scale height is constant is hard to understand, since current theory predicts that the vertical profile shape is determined by heating by molecular clouds, of which the distribution changes considerably as a function of radius [17]. Other mechanisms will have to be invoked (see [14]).

For the sample described above we have analysed their vertical exponential scale heights, and their radial dependency. These results are shown for 6 galaxies of different types in Fig. 4a. It shows that for every galaxy the scale length increases, and that the increase is strongest for the earliest-type galaxies. In the area of the bulge the scale heights are smallest. One might

think that this effects is caused by the fact that the bulge contaminates the fit to the disc. This is incorrect - for example: Barnaby & Thronson [6] find for NGC 5907 that the scale height is lower between $-100''$ and $+100''$ than in the rest of the galaxy. They say that this due to their bulge contamination. However, as can be seen from their Fig. 4, between $\pm 50''$ and $\pm 100''$ the bulge contribution is negligible. We find that in none of the objects bulge contamination is responsible for the increase of scale height with radius.

As a following step we have fitted the radial change in scale height between 2 and 4 I -band scale lengths. We find (Fig. 4b) that in all our galaxies scale heights increase radially or remain constant. For morphological types larger than 2 the change is very small, but for earlier types the gradients are considerable.

What can we conclude? It has been known for a long time time that S0 galaxies show the presence of a thick disc component, much stronger than the thick disc in our Galaxy ([11, 27]). They also see that this thick disc starts dominating the light in the outer parts, indicating that its scale *length* is larger than the scale length of the main disc. If one fits only one exponential to the vertical distribution, one will see an increase of the scale height with radius. For the later types van der Kruit & Searle [31] note that the late type edge-on galaxies NGC 4244 and NGC 5907 have larger scale heights in their outermost profiles. It seems that spiral galaxies in general show the presence of thick discs with scale lengths larger than those of the main disc, and that the relative importance of the thick disc goes down strongly towards later types. The origin of the thick discs is not very clear. It seems that the presence of the thick disc is determined completely by the morphological type, or maybe the gas fraction of the galaxy. The process that makes discs of S0 galaxies in general thicker than discs of later type galaxies, e.g. interactions, would then also be responsible for the formation of the thick disc. At the surface brightness that we are observing these galaxies are still very symmetric (see Fig. 4a), so we are not looking at the presence of warps here.

2.3 Galactic bulges

We end this section on the morphology of spiral galaxies by summarising some recent results for galactic bulges. For years the $r^{1/4}$ profile [12] has been extremely popular in modeling surface brightness of bulges (e.g. [19]), although often significant deviations were observed, and no conclusive paper had appeared showing that this law was fitting well in general for bulges. Presumably, people liked to use it, to emphasise the evolutionary link with elliptical galaxies, which are in general well fitted by it (e.g. [10]). Andredakis & Sanders [3] showed that the photometric data of Kent [19] could be fitted with exponential bulges just as well as with $r^{1/4}$ bulge profiles, with much less internal scatter in the fitted parameters. Using higher quality data, Andredakis *et al.* [2] subsequently showed that bulges of *early-type* spirals could be fitted in general well by the $r^{1/4}$ law, while *late-type* bulges looked much more exponential. Parametrising the surface brightness law by:

$$K(r) = K_0 \exp(-r/r_0)^{1/n} \quad (3)$$

they found that the fitted n correlated very well with morphological type, or with bulge-to-disc ratio. A likely cause of the relation is that the interaction of bulge and disc had altered the bulge profile in the outer parts. Recently however, the results from HST have again complicated this picture. Apart from an exponential or $r^{1/4}$ bulge Phillips *et al.* [24] found that many bulges of morphological type earlier than Sc also had an unresolved stellar nucleus with very high surface brightness. More analysis is needed for us to understand these cusps and the interaction between the cusps and the rest of the galaxy.

3 The colour-luminosity relation in spiral galaxies

To understand the formation and evolution of galaxies it is very important to study their scaling relations, i.e. relations between their fundamental quantities like mass, diameter, surface brightness etc. For early-type galaxies there is a tight relation between colour and luminosity: Bower *et al.* [7, 8] determined that for the colour-magnitude relations in $U - V$ and in $V - K$ the scatter in both the Virgo and the Coma cluster was comparable to the observational scatter, and that the CM relations had the same slope. Early-type galaxies generally have very little star formation and extinction by dust, so their colour directly reflects their stellar populations. Bower *et al.* interpret the colour-magnitude relation as a relation between mass and metallicity, and in this interpretation all early-type galaxies in Coma and Virgo must have formed at approximately the same time. Ellis *et al.* [13] recently also found tight CM relations in 3 distant clusters, showing that the early-type galaxies in those clusters also must have formed at the same time, or are very old.

It is hard to derive similar constraints for spiral galaxies, mainly because of the fact that recent star formation and extinction are responsible for considerable scatter. It would however be very interesting to know more about CM relations for spirals, especially to study their star formation histories. When studying an infrared CM diagram one could study the older stellar populations in spirals, and find out whether they were formed at the same time as the early-type galaxies. The optical-infrared CM relation was studied in detail by Tully *et al.* [28], who claimed that spiral galaxies had a different CM relation than lenticulars, while both relations had very little scatter. They claimed that the difference could be explained by the presence of more star formation in late type spirals, affecting especially the blue light, and hence the $B - H$ colour. Their result, except for the fact that it was very hard to interpret, since they used the hybrid $B_T - H_{-0.5}$ colour, was not confirmed by aperture photometry of Mobasher *et al.* [22], who found that early and late type galaxies had more or less the same CM-relation in $B - K$ and $J - K$, although the scatter was much smaller in $J - K$, but still considerable. Almost certainly Tully *et al.*'s data have not been corrected enough for extinction, since some of his S0 galaxies were redder than the reddest ellipticals. This is in disagreement with Balcells & Peletier [5], who showed that dustfree colours of bulges of S0's are always bluer or have the same colour as ellipticals of the same luminosity. These authors also don't reproduce the gap in colour that Tully *et al.* find between S0's and later type spirals.

Nowadays, using infrared arrays, it is possible to derive CM-relations with a much smaller scatter, by measuring for each galaxy the colour in areas where extinction is thought not to be important. We will discuss here the $I - K$ vs. M_K relation for a sample of spiral galaxies consisting of two subsamples: 1) a sample of early-type spiral galaxies with inclination $> 50^\circ$, where the bulge is unobscured by the disc (from [23]). Of this sample we only use the galaxies of type Sa and earlier, to avoid dust-affected colours. For this sample we took the bulge colour at $5''$ from the center on the minor axis. 2) a sample of edge-on ($i \geq 87^\circ$) galaxies, from de Grijs *et al.* [15]). Colour maps and profiles show that the colour in the vertically outer parts is constant, and that the colour profiles are featureless and symmetric compared to the other side of the galaxy. For this reason we assumed that all the extinction is concentrated near the plane of the galaxy, and that the outer regions are dustfree. The colours were determined on both side of the plane on the minor axis in the area where the colour is constant (appr. 1-3 scale heights).

We have plotted our CM relation in Fig. 5a, together with Bower's points. Also drawn are two least-squares fits. The fit for the spirals was determined taking into account the errors in both directions. We find that the scatter in $I - K$ amounts to 0.12 mag, but if one takes into account the observational uncertainty, the scatter goes down to 0.07 mag, i.e. much smaller

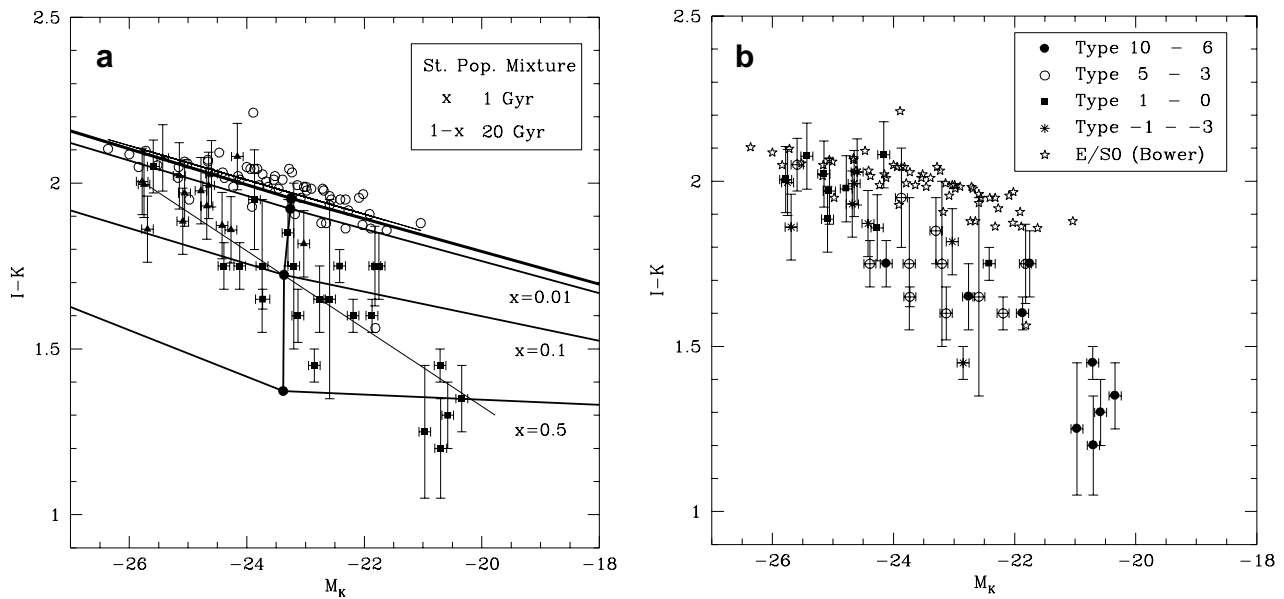


Figure 5: **a:** The $I-K$ vs. M_K relation. Filled squares indicate the data of de Grijs *et al.* [15], filled triangles the early-type spirals of type ≤ 1 of Peletier & Balcells [23], and open circles the early-type galaxies of Bower *et al.* [7]. The thick lines connect some stellar population models, as discussed in the text. The thin lines are least-squares fit to resp. the early and late-type galaxies. **b:** The same data, now as a function of morphological type.

than was published before. Fig. 1 shows that

- Spiral galaxies have a tight colour-magnitude relation, with a scatter that is slightly larger than can be explained by observational uncertainties.
- The relation has a different slope than the CM relation for ellipticals, and there is an area between the two relation in which very few galaxies are found.

We see two ways to explain these observational findings. The first is to revert to the explanation of Tully *et al.* [28], who state that there is more star formation in spirals than in ellipticals. The blue light from the stars affects $I-K$, although much less than e.g. $B-K$, and leaves M_K almost unaffected. As an example we have drawn some two-burst stellar population models using Bruzual & Charlot [9] in Fig. 5a of 20 and 1 Gyr, where the younger component consists of a fraction x (50, 10 and 1%) of the mass. We see that for the smallest galaxies about 50% of the stars should be young, while for brighter galaxies this number should be 10%, going down more for the earliest type spirals. We could also take different stellar population models (for example 20 and 0.5 Gyr, exponentially decreasing or continuous star formation), and find other numbers. The most interesting aspect of this work is that we find that the scatter in the CM relation for spirals is so very small, and that there is a gap between spirals and ellipticals. It means that the current star formation in a spiral galaxy is determined by its size, morphological type or luminosity, not by its environment, interactions or evolution. It also means that for example dwarf ellipticals are very different from dwarf irregular galaxies.

There is however another explanation, namely that in $I-K$ spirals have the same CM relation as ellipticals, but that the difference between the two relations is purely caused by stellar population gradients in the vertical direction in the spirals. In this case we still would have to explain why the scatter between the spirals is so very small. Are vertical colour gradients

important? At the moment we don't know very well what the average vertical colour gradients in spirals look like, but we can make a quantitative estimate. For the sample of Peletier & Balcells [23] the average $I - K$ colour gradient of the bulge was found to be $\Delta(I - K)/\Delta(\log r) = -0.19$, with a RMS scatter of 0.09. The radial disc-gradients also are not zero: the average $\Delta(I - K)/(\text{K-band scale length})$ here is -0.10 , with a scatter of 0.09. And furthermore bulges are slightly redder than discs (on the average 0.07 mag). Fisher *et al.* [16] also show that vertical metallicity gradients in bulges, as derived from the Mg_2 index, are much larger than radial disc gradients. In this paper the colour of the edge-on galaxies was determined between approx. 1 and 3 scale heights, and from the previous numbers it seems possible that the difference between the colour of the total galaxy and our colour is as large as $\Delta(I - K) = 0.3$ (or 0.40 in $[\text{Fe}/\text{H}]$), although a detailed study will have to confirm whether this is really the case. The fact that the scatter between spirals is so small indicates in this interpretation that all spirals have similar colour gradients. Since vertical colour gradients would be larger than radial gradients, galaxies not seen edge-on, even without extinction, would create a larger scatter in the CM relation for spirals.

From the analysis presented in this paper we draw the following conclusions:

- We have determined a dust-free CM relation for spiral galaxies, by measuring $I - K$ colours in edge-on galaxies above the plane. We find that the scatter in this relation is small and approximately as large as can be accounted for by observational uncertainties. The slope of the IR CM-relation is larger for spirals than for elliptical galaxies.
- We have two possible explanations. First, the difference could be caused by vertical colour gradients in spiral galaxies. In that case these gradients should be similar from galaxy to galaxy, have an average size of about 0.15 dex in $[\text{Fe}/\text{H}]$ per scale height, and increase for later galaxy types. Spirals and ellipticals would have the same colour-magnitude diagram, indicating that the mass of a galaxy determines completely the old stellar populations, predicting a very low scatter in the IR Tully-Fisher law. The other possibility would be that spirals and ellipticals have different CM relations. The difference would be caused by current star formation, which has to be present in all spirals. The amount of current star formation would depend only on the galaxy luminosity, and not on environment.

Acknowledgements. We thank P.C. van der Kruit for useful discussions, and the conference organisers for organising a very pleasant and interesting meeting.

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